The Association
of
Engineering and Shipbuilding
Draughtsmen.

Considerations in the Choice of Working Stresses and Factors of Safety.

By J. LEWIN, A.M.I.Mech.E.

Published by The Association of Engineering and Shipbuilding Draughtsmen,
Onslow Hall, Little Green, Richmond, Surrey.

SESSION 1951-52.

Price 3/-

Printed by Milne, Tannahill & Methven, Ltd. (T.U.), 12-14 Mill Street, Perth.

ADVICE TO INTENDING AUTHORS OF A.E.S.D. PRINTED PAMPHLETS.

Pamphlets submitted to the National Technical Sub-Committee for consideration with a view to publication in this series should not exceed 10,000 to 15,000 words and about 20 illustrations. The aim should be the presentation of the subject clearly and concisely, avoiding digressions and redundance. Manuscripts are to be written in the third person.

Drawings for illustrations should be done either on a good plain white paper or tracing cloth, deep black Indian ink being used. For ordinary purposes they should be made about one-and-a-half times the intended finished size, and it should be arranged that wherever possible these shall not be greater than a single full page of the pamphlet, as folded pages are objectionable, although, upon occasion, unavoidable. Where drawings are made larger, involving a greater reduction, the lines should be made slightly heavier and the printing rather larger than normal, as the greater reduction tends to make the lines appear faint and the printing excessively small in the reproduction. In the case of charts or curves set out on squared paper, either all the squares should be inked in, or the chart or curve should be retraced and the requisite squares inked in. Figures should be as self-evident as possible. Data should be presented in graphical form. Extensive tabular matter, if unavoidable, should be made into appendices.

Authors of pamphlets are requested to adhere to the standard symbols of the British Standards Institution, where lists of such standard symbols have been issued, as in the case of the electrical and other industries, and also to the *British Standard Engineering Symbols and Abbreviations*, No. 560, published by the B.S.I. in 1934 at 5/-. Attention might also be given to mathematical notation, where alternative methods exist, to ensure the minimum trouble in setting up by the printer.

The value of the pamphlet will be enhanced by stating where further information on the subject can be obtained. This should be given in the form of footnotes or a bibliography, including the name and initials of the author, title, publisher, and year of publication. When periodicals are referred to, volume and page also should be given. References should be checked carefully.

Manuscripts, in the first instance, should be submitted to the Editor, *The Draughtsman*, Onslow Hall, Little Green, Richmond, Surrey.

For Pamphlets, a grant of £20 will be made to the Author, but special consideration will be given in the case of much larger pamphlets which may involve more than the usual amount of preparation.

The Publishers accept no responsibility for the formulae or opinions expressed in their Technical Publications.

The Association
of
Engineering and Shipbuilding
Draughtsmen.

Considerations in the Choice of Working Stresses and Factors of Safety.

By J. LEWIN, A.M.I.Mech.E.

Published by The Association of Engineering and Shipbuilding Draughtsmen,
Onslow Hall, Little Green, Richmond, Surrey.

SESSION 1951-52.

9/53

CONTENTS

Page

Introduction - - - - - - - - 5

PART I.

Factors of Safety and Load Factors - - - - - 7
Some Physical Properties of Metals and Alloys - - 9
Design Criteria in Ductile Materials under Simple Stress 11
Elongation and Ductility - - - - - - 13
Stress Distribution - - - - - - - 15
Plastic Behaviour in Structures as a Basis of Design - 16
Design Criteria in Brittle Materials - - - - 18
Criteria of Failure under Combined Stresses - - 21
Elastic Instability - - - - - - - 23
Effects of Overstraining - - - - - - 25
Creep - - - - - - - - 27

PART II.

The Choice of Working Stresses under Dynamic Loading 28
Impact Values - - - - - - - 31
Alternating Stresses - - - - - - 32
Combined Alternating Bending and Shearing Stress - 36
Combined Steady and Variable Stress - - - 37
Combination of Static and Variable Bending and Torsional Stresses in Ductile Materials - - - - - 39
Stress Concentrations under Fatigue Conditions - - 41

APPENDIX I.

Values of Rupture Factor for Cast Iron - - - 44

APPENDIX II.

Factors of Stress Concentration for Elastic Stress 45

INTRODUCTION.

The choice of working stresses in the dimensional calculations of machine members is one of the most important problems encountered. It can be defined as an analysis of the margin required between service conditions and failure.

The aim has been to set out some of the more important considerations governing the choice of the margin of safety and hence working stresses rather than to present formulae and numerica values. Considerations have been discussed mainly from the point of view of the behaviour of bodies under stress, others that can be assessed only from experience fall outside the scope of these pages.

Some information which the author hopes will be of use in practical design work has been appended either in tabular or graphical form.

The subject matter has been divided into two parts which although interdependent provide a convenient separation. The first deals with general design considerations and those pertaining to static loads, the second with problems specific to alternating or impact loads.

CONSIDERATIONS IN THE CHOICE OF WORKING STRESSES AND FACTORS OF SAFETY.

By J. Lewin, A.M.I.Mech.E.

PART I.

FACTORS OF SAFETY AND LOAD FACTORS.

Two methods can be adopted to arrive at a design stress, that is the stress used in mathematical determination of the required size.

(1) If the stress corresponding to failure which a material can withstand is denoted by f, the design stress is given by f/n, where n is called a factor of safety.

Failure may mean rupture of a machine member or collapse of a structure, or permanent deformation making the part unfit for service before the end of its normal life.

(2) If the external load is multiplied by a factor N and the failure stress is used as the criterion of design, 'N' is called a load factor.

For most conditions both methods will give identical results. There are, however, cases when there is a considerable divergence. A specific example in the text will illustrate this.

The reciprocal of the factor of safety is referred to as the factor of utilisation. It is more convenient mathematically and also somewhat more logical to use a factor of utilisation; however, because of the wide usage of the factor of safety it was considered advisable to retain its use in these pages.

The factor of safety or the load factor respectively is used to denote the ratio of failure to service value for any condition in which such factor is provided as safeguard.

A grouping of considerations that will lead to the adoption of a particular factor has been attempted below.

(1) The possible consequences of failure.

If failure is attended by extensive damage or possible loss of life, the margin of safety will obviously be chosen more stringently than where failure can be made good by quick replacement without serious disruption of working routine.

(2) Unintentional overloads.

(3) Reliability of material and knowledge of its physical properties.

(4) The method of load application, whether steady, fluctuating, subject to impact or combination of some or all of these methods of loading.

(5) Manner of supports.

In structures subsidence of foundations can cause severe stresses to be set up even before external loads are applied. In machines or machine members the manner of support and its rigidity determine to some extent the resistance to shock loads.

(6) Limits and accuracy of mathematical analysis.

Most formulae in stress analysis are based on certain assumptions as to properties of materials, regularities of form and boundary conditions, and they are derived by mathematical procedures that often involve further approximations. The choice of the factor of safety has, therefore, to be made with regard to the practical realisation of theoretical postulates. Where these cannot be held to apply and where load combinations do not permit accurate stress analysis the margin of error must be covered by a greater factor of safety.

(7) Exposure to corrosion and other chemical action.

A specific example of chemical action is the electrolytic action of some dissimilar metals in contact, which causes surface deterioration.

(8) Effects of temperature at which the member is to operate.

(9) Errors in workmanship.

(10) Lessening of original dimensions by legitimate wear.

(11) The estimated working life of the part.

Some of these factors can be assessed numerically with fair accuracy, others cannot be computed, but an understanding of the behaviour of bodies under stress, coupled with past experience may often be of help in the exercise of judgment.

The penalties for too stringent a factor of safety will be apparent when it is remembered that in a structure of given material, it will cause an increase in stresses due to the "self-load" and require heavier foundations. In machines this applies equally as well as an increased demand in accelerating and running power.

For identical conditions of stress, widely differing factors of safety might be chosen, either from considerations of operating conditions or just expediency.

SOME PHYSICAL PROPERTIES OF METALS AND ALLOYS.

Fig. 1 shows a stress-strain diagram for a mild steel specimen which can be taken as typical for a group of ductile alloy steels. OA is a straight line representing the stage during which Hook's law may be said to apply, stress being proportional to strain. A is the limit of proportionality; the line beginning to curve at this point. Slightly below A on the straight line, but for practical purposes coincident with it, is the elastic limit. Stress and strain cease to be reversible, permanent set being observable when unloading, beyond this point. B marks the yield stress when straining takes place without any increase in load. Careful experiments have shown the existence of an upper yield stress C, which is, however, of little practical significance. BD is the plastic stage. At D strain hardening begins and there is a recovery during the ductile stage until at E the maximum load is reached. EF denotes the local drawing out during which the load is reduced until the breaking point F is reached.

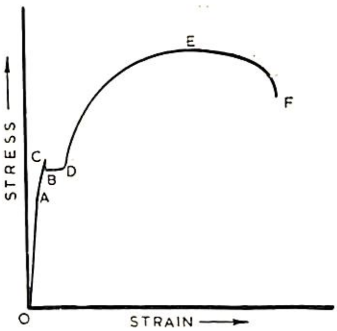


Fig. 1—Stress-Strain Diagram for Mild Steel.

E is called variously the ultimate stress, calculated on the original cross-sectional area, ultimate strength or tenacity.

Fig. 2 shows a typical stress-strain diagram for a high strength alloy steel. There is no sudden increase in strain as in Fig. 1, but the extension goes on with increase in load until local drawing out or "waisting," as it is sometimes called, takes place. In lieu of a well defined yield stress, a limit called the proof stress is used to denote a percentage of permanent set.

The 0·2% proof stress generally in use with reference to steels is the nominal stress that will produce 0·2% permanent set of the length between gauge points.

For aluminium and magnesium alloys, which also exhibit no well-defined yield stress a 0·1% proof stress is commonly used.

In high strength alloys the limit of proportionality is usually found to be lower than in mild steel considered as a percentage of the yield or proof stress.

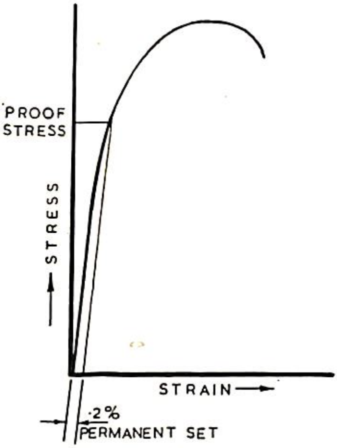


Fig. 2—Stress-Strain Diagram for a High Strength Alloy Steel.

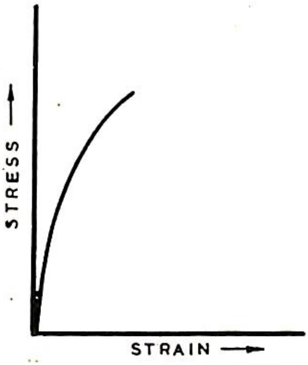


Fig. 3—Stress-Strain Diagram for a Brittle Material.

Fig. 3 finally shows a typical stress-strain diagram for a brittle material. There is no plastic stage and very little elongation and no local drawing out, that is, reduction in area. Departure from proportionality takes place early in the stress-strain diagram.

The dividing line between the two classes of metals has been suggested as 5% ultimate elongation.

BIBLIOGRAPHY.

Batson, R. G., and Hyde, J. H., *Mechanical Testing* (Chapman & Hall), 1931, Vol. 1, *Testing of Materials of Construction*, p. 30.

Docherty, J. G., and Thorne, F. W., 1931, *Engineering*, Vol. 132, p. 295, *The Phenomena of Tensile Yield in Mild Steel and Iron.*

Foster, P. F., *The Mechanical Testing of Metals and Alloys* (Pitman), 1938, p. 73.

Roberts, A. M., *Mechanical Tests for Engineering Materials* (A.E.S.D.), 1948-49, p. 14.

Salmon, E. H., *Materials and Structures* (Longmans, Green), 1931, Vol. 1, *The Elasticity and Strength of Materials*, p. 431.

DESIGN CRITERIA IN DUCTILE MATERIALS UNDER SIMPLE STRESS.

In the design of machine members or structural parts in ductile materials the yield stress or proof stress respectively is generally accepted as the criterion of design. Working stresses are then arrived at by applying a factor of safety. Under normal service conditions the working stresses remain within the elastic limit of the material as the deformation attendant on reaching the yield stress is assumed to make the member unserviceable.

The use of a proof stress is to some extent arbitrary. In high strength steels there is often an early departure from proportionality, which may be as high as 40% of the proof stress. The use of the limit of proportionality does not result in a proper utilisation of the strength of the material.

It may be recalled that in a specimen under direct uniaxial tensile stress, there is a shear stress along any plane oblique to the line of action of the tensile stress. If f_t is the direct tensile stress, the magnitude of the shear stress is given by $f_t \sin \theta \cos \theta$ or $(f_t \sin 2\theta)/2$ and evidently reaches a maximum value $f_t/2$ at a plane inclined at 45%. (Fig. 4).

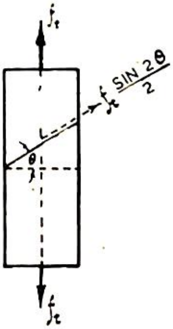


Fig. 4.

If the shear strength of a material is less than half its tensile strength, then the material will fail by shear when subjected to a tensile load. Similarly if the applied stress is compressive and elastic instability (buckling) is ruled out, the material will fail by shear.

At the onset of plastic flow elastic breakdown is observable on ductile tensile test specimens by the appearance of slip or Lüder lines, inclined at an angle to the principal axis, indicating that the maximum shear stress is the criterion of elastic failure. Plastic flow in ductile ferrous materials at yield conditions commences at particular places in the specimen, not over its whole length, until strain-hardening commences. It then continues over contiguous sections until the whole specimen has been strain-hardened to the same extent. Recovery is then complete and further deformation and hardening occurs uniformly over the whole specimen, and continues thus until local drawing out begins. During this last stage the material exhibits brittle characteristics.

Since the shear stress induced by a direct tensile stress causes elastic breakdown it follows that if a torsional stress is applied, failure will occur when half the yield stress in tension is exceeded.

The maximum shear stress criterion of failure in ductile materials is not consistent with observations that the shear stress to cause failure in simple torsion is higher than that required to produce plastic deformation in simple tension.

The shear strain energy criterion (due to Huber, Mises & Hencky) does permit closer prediction of failures under various conditions of stress and it has been generally adopted. According to this theory, which will be more fully discussed under the chapter of combined stresses, elastic breakdown commences when the shear strain energy exceeds a particular limit.

Based on the Huber, Mises & Hencky theory and other interpretations (Cox & Sopwith), theoretical values of the shear stresses necessary to initiate yielding under simple torsion give values of the order of 15% higher than that of half the yield stress in tension. Thus the stress at which elastic breakdown occurs in torsion is 0·575 of the stress causing failure in direct tension. There has been good experimental evidence in support of the shear energy criterion (see, for instance, Morrison 1948).

It follows that the application in practice of half the yield stress in direct tension as a criterion in the design of members in torsion leads to dimensions somewhat larger than are consistent with a given factor of safety, but it is a safe basis of calculations.

The author's observations, however, have led him to the conclusion that the ratio of yield stress in tension to that in torsion is by no means constant.

Summing up rules for working stresses for ductile materials under steady loads :—

(*a*) Ductile materials are said to suffer elastic failure if the direct tensile, compressive or bending stress reaches the yield stress

$$n = f_y/f \qquad (1)$$

where n is a factor of safety, f_y is the yield point stress, and f is either a direct tensile stress

or a direct compressive,

or an extreme fibre stress due to bending.

(*b*) Ductile materials are said to suffer failure if the direct shear or torsional stress becomes equal to half the yield stress in direct tension (theory of Coulomb and Guest)

$$n = f_y/2q \qquad (2)$$

where q is either a direct shear stress
or a torsional stress.

or

Ductile materials are said to suffer failure if the shear strain energy exceeds a particular limit (theory of Huber, Mises and Hencky).

$$n = f_y/\sqrt{3}q \qquad (3)$$

The two preceding statements are rational, that is, they have a theoretical basis.

BIBLIOGRAPHY.

Morrison, J. L. M., 1948, Proc.I.Mech.E., Vol. 159, p. 81, *The Criterion of 'Yield' of Gun Steels.*

Morrison, J. L. M., and Shepherd, W. M., 1950, Proc.I.Mech.E., Vol. 163, p. 1, *An Experimental Investigation of Plastic Stress-Strain Relations.*

Cox, H. L., and Sopwith, D. G., 1937, Proc. Phys. Soc., Vol. 49, p. 134,.

Warnock, F. V., *Strength of Materials* (Pitman), 1945, p. 18.

ELONGATION AND DUCTILITY.

If the total elongation of a specimen under tensile stress be expressed as a percentage of the original gauge length, a measure of ductility is obtained. The experimental value of the percentage elongation is affected by the choice of the gauge length. In order to obtain consistent values, this length is defined in this country as four times square root of cross-sectional area for specification purposes.

The work done in stressing a specimen is given by the area under the stress-strain diagram, which is, therefore, an index of energy. Up to the limit of proportionality this work is stored in the body as strain energy and is recoverable. Above that value the energy is used in permanently deforming the body and cannot be recovered. An inspection of Fig. 1-3 will show that the energy absorption capacity to rupture of a metal or alloy is greater for a high value of percentage elongation. In this respect mild steel exhibits most favourable characteristics because of its extensive plastic deformation stage.

In a machine member or structure maximum stresses are rarely reached simultaneously at every section. If elastic stresses are exceeded locally, either due to overload, unforeseen impact loads or soil subsidence, the material on reaching the yield stress will cause local plastic flow with consequent redistribution of stress and give warning due to visible deformation.

The energy absorption capacity of steels at shock loads and the additional load that can be carried until rupture is a valuable safeguard where collapse would be attended by extensive damage.

One might, therefore, extend the previously discussed design concept, which can be defined as maintaining permanence of dimensions by adding a further criterion, freedom from failure at any point.

A comparison of the properties of low carbon alloy steels shows that for a given ultimate strength an increase in the yield stress is often obtained at the expense of ductility.

The ratio of the ultimate strength to the elastic limit stress (usually taken in practice as the yield stress) is called the elastic ratio. In view of the foregoing, some designers feel that elastic ratio values give an indication of the effective ductility of the material. If this is granted, the overall factor of safety must be made a function of the elastic ratio. This overall factor of safety then consists of the real margin of safety, 'n' and the elastic ratio 'e.' It is applied to the ultimate strength of the metal to obtain the working stress.

Thus overall factor of safety $= n \times e$

$$\text{and } f = \frac{f_u}{n \times e}$$

where f is the design or working stress and f_u is the ultimate strength of the material.

A specified value of the elastic ratio will determine the choice of the material.

The elastic ratio cannot altogether be relied upon as an index of ductility. It can be said though to be a measure of the additional stress that can be carried until rupture, if one bears in mind that stresses calculated by the ordinary formulae of mechanics (*i.e.*, flexure, torsion, etc.) postulate a linear stress-strain relationship and that failure stresses calculated on this basis will be fictitious.

BIBLIOGRAPHY.

Batson, R. G., and Hyde, J. H., *Mechanical Testing* (Chapman & Hall), 1931, Vol. 1, *Testing of Materials of Construction*, p. 98.

Foster, P. F., *The Mechanical Testing of Metals and Alloys* (Pitman), 1938, p. 75.

Roberts, A. M., *Mechanical Tests for Engineering Materials* (A.E.S.D.), 1948-49, p. 23.

Salmon, E. H., *Materials and Structures* (Longmans, Green), 1931, Vol. 1, *The Elasticity and Strength of Materials*, p. 439.

STRESS DISTRIBUTION.

Elastic stress distribution across a section may be nominally uniform as in direct axial tension or compression or may vary in some regular manner, as for instance in a beam subject to flexure. In practice, the actual distribution of stress is determined by a number of considerations, the most important being the shape of the member.

A slight change in section is sufficient to cause some departure from uniform stress distribution. A rapid increase in intensity of stress takes place if the variation is abrupt. Generally, the sharper the re-entrant angle, the greater the intensity of stress, called a stress concentration. The same effect is caused by any local irregularities of form, such as holes, notches or rapid change of form, such as a corner.

The maximum elastic stress caused by such a stress raiser can be ascertained either by mathematical computation or experimentally by direct measurements by strain gauges, or by the use of polarized light in photoelastic analysis.

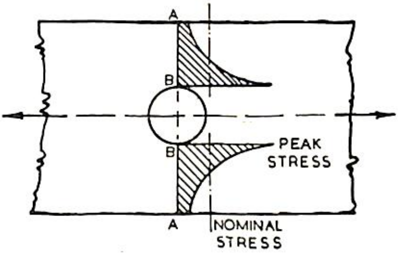


Fig. 5—Stress Distribution in a Rectangular Bar containing a Circular Hole.

Fig. 5 shows a bar with a circular hole subjected to pure tension. Because of the presence of that hole the stress distribution over the cross-section AB will not be uniform. There is a peak stress at B, which is about three times as large as the nominal stress obtained by dividing the tensile force by the net area of the cross-section.

The ratio of the true maximum stress to the stress calculated assuming uniform distribution (*i.e.*, by the ordinary formulae of mechanics, flexure, torsion, etc.) is called the stress concentration factor. It is a measure by which the nominal stress has been exceeded.

Fig. 6 shows the stress concentration factors for a particular type of stress raiser, in this case fillets in tension.

The occurrence of stress concentrations is characteristic of elastic behaviour. The high local stress on reaching the elastic limit causes local plastic flow of the material at the point of stress

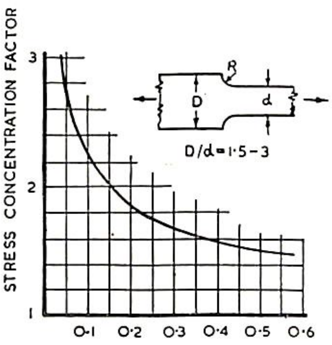


Fig. 6—Stress Concentration Factor for Fillets in Tension.

concentration with a consequent readjustment of the stresses without yielding or failure of the whole member. Some stress relief takes place even in relatively brittle materials. Because of this readjustment high local stresses are of less importance under static loading than consideration of elastic stresses alone would indicate. Generally one can say that for ductile materials the mean stress may be used to control the design.

BIBLIOGRAPHY.

Timoshenko, S., *Theory of Elasticity*, Eng. Societies Monograph (McGraw-Hill), 1934.

Timoshenko, S., 1947, Proc.I.Mech.E., Vol. 157, p. 163, *Stress Concentration and Fatigue Failures.*

PLASTIC BEHAVIOUR IN STRUCTURES AS A BASIS OF DESIGN.

In the use of ductile materials, local yielding may not cause sufficient deformation to render the member unusable. In redundant structures, local yielding in flexure can cause redistribution of moments and the ultimate load carrying capacity may be in excess of that required for the most highly stressed part to reach the yield point.

In order to predict the behaviour of a member under flexure beyond the elastic range the theory of plastic bending must be used.

Consider the stress distribution in flexure of a rectangular mild steel beam shown in Fig. 7 (*a*). If the applied bending moment be increased so that the extreme fibre stress exceeds the yield strength, the stress in the fibre will remain constant and due to its deformation in the plastic stage at continuing increasing bending moment, the stress distribution will be as in Fig. 7 (*b*), which corresponds to a moment of resistance somewhat greater than the yield moment.

The maximum moment of resistance is obtained when the whole section has been strained beyond the yield point, Fig. 7 (*c*), its value then being referred to as the full plastic moment.

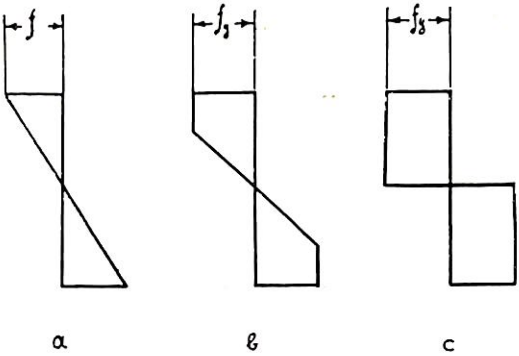


Fig. 7.

The ratio of M_p (plastic moment) to M_y (yield moment) is a measure of the increase in load carrying capacity, and is referred to as the "shape factor."

For a rectangular beam bent about an axis of symmetry ;

M_y = yield stress × elastic section modulus

$$M_y = f_y \ bd^2/6 \tag{5}$$

M_p = yield stress × first moment of area

$$M_p = f_y \ bd^2/4 \tag{6}$$

Therefore for a rectangular beam, the shape factor is 1·5. For most standard rolled I beams (B.S. 4) bent about the major axis it is 1·15 approximately.

When the full plastic moment has been reached, as illustrated by the stress distribution in Fig. 5 (*c*), the deflections for a simply supported beam become infinitely large and collapse is said to occur. The corresponding load is greater than at the onset of yield, the difference is given by the shape factor. If, however, the ends of the beams are restrained in direction, the beam will now act as two cantilevers hinged at the middle and collapse cannot occur until the full plastic moment has been reached at the restrained ends as well.

In a structure with one redundancy the full plastic moment must be developed at two sections.

Where there are two redundancies three sections must develop full plastic moment and so on.

The application of the plastic theory of bending in the design of structures is a far more rational one as the criterion is collapse. Its application can lead to considerable economies in material.

Investigators, notably J. F. Baker and J. Heyman, have been able to predict with fair accuracy the behaviour of some types of portal frames stressed into the plastic range.

The difficulty inherent in the application of the plastic theory of bending to the design of structures lies in determining a permissible deformation. This may be governed by such considerations as the distortion of windows and the cracking of plaster work in buildings. For this reason plastic design has at present been principally developed for the use of portal type sheds.

BIBLIOGRAPHY.

Baker, J. F., October, 1949, Journal I.Struct.E., Vol. 27, p. 397, *The Design of Steel Frames.*

Baker, J. F., and Heyman, J., June, 1950, Journal I.Struct.E., Vol. 28, p. 139, *Tests on Miniature Portal Frames.*

Baker, J. F., 1948-49, Journal I.C.E., No. 3, p. 188, *A Review of Recent Investigations into the Behaviour of Steel Frames in the Plastic Range.*

Baker, J. F., 1935-36, Journal I.C.E., Vol. 3, p. 127, *The Rational Design of Steel Building Frames.*

Baker, J. F., and Roderick, J. W., N.E. Coast Inst. Engineers and Shipbuilders, Newcastle-upon-Tyne, 25th April, 1941, *Plastic Theory—Its Application to Design.*

Roderick, J. W., Phil. Mag. Ser. 7, Vol. 39, p. 529, *Theory of Plasticity—Elements of Simple Theory.*

DESIGN CRITERIA IN BRITTLE MATERIALS.

The deformation to rupture is comparatively small in brittle materials. One cannot, therefore, consider such materials to have definitely failed until they have broken, and this takes place either through a tensile fracture, that is normal to the plane of maximum tensile stress, when that stress reaches the ultimate strength, or through an apparent shear fracture when the maximum compressive stress reaches a certain value. Fracture occurs at a plane oblique to the maximum compressive stress, but not as a rule at an angle of 45° to it, the plane of maximum shear stress.

It can be shown that a torsional stress is equivalent to a tensile and a compressive stress, each equal in intensity to the shear stress and each making an angle of 45° with the direction of the shear stress. In a typical brittle material like cast iron where the resistance to tension is less than the resistance to direct shear, the material fractures in tension along a spiral line, when a torsional stress is applied.

In calculating the stress at rupture, also called the modulus of rupture in bending or torsion respectively, the application of the ordinary formulae of flexure or torsion does not result in a true stress. The stress distribution as calculated by the above formulae is strictly valid only up to the elastic limit. The failure stress or rupture modulus can, however, be used to predict the strength of a similar member of the same material.

An accurate assessment requires a knowledge of the following factors.

In beams subjected to bending :—

(1) The effect of the span-depth ratio.
(2) The scale (size).
(3) The shape of the cross-section of the beam, which is taken into account by the form factor.
(4) The manner of loading.
(5) The material (cast iron, bakelite, plaster, etc.).

Similar considerations apply to torsion members, except (1). The rupture factor provides a means of predicting the disparity between the nominal maximum tensile stress at rupture as calculated by the appropriate elastic theories and the failure stress, as determined by subjecting the material to a tensile test. The rupture factor is the ratio of these stresses. If a member of given form, size and material, loaded and supported in a given manner has been found to rupture at a certain load, corresponding to a tensile stress f_l calculated by an elastic formula, and if f_u is the ultimate stress at which a standard tensile test specimen fractures, f_l/f_u is the rupture factor.

In Appendix I, some values of the rupture factor have been tabulated.

For ductile materials in bending a redistribution of stress takes place when the extreme fibre stress reaches the yield stress, and the load carrying capacity at failure has been shown to be greater than that at elastic breakdown. In brittle materials a similar process, much less in degree, takes place and the rupture factor (as will be seen from Appendix I) for a beam in flexure is well above unity. Generally, the smaller the part of the member subjected to high stress, the larger the rupture factor. Thus a circular section used as a beam has a higher modulus of rupture than a rectangular one. The former experiences the extreme fibre stress over a very short section of its circumference, the latter along its whole breadth.

As a rule the more brittle the material, the more nearly all rupture factors approach unity. Since percentage elongation is a measure of brittleness, it is an indication of the degree of redistribution of stress that can be expected after the proportional limit stress has been exceeded. With very brittle materials, very little redistribution will take place and rupture conditions can be predicted with fair accuracy by formulae based on elastic stresses. There are, however, many exceptions to this rule.

The effect of the span to depth ratio in cast iron is fairly small, the strength reduction being about 10% for ratios from 5 - 30.

When the maximum bending moment occurs at but one section, as in the case of a single concentrated load, the stress at failure is higher than when the maximum bending moment extends over a considerable part of the span. The last condition is realised when there are two identical loads equidistant from the supports. The disparity decreases as the span to depth ratio increases.

The effect of form is sometimes expressed by a form or shape factor, but can equally well be represented by the rupture factor. The form factor is also of importance in the flexure of ductile members, and it is felt that it is probably better to restrict its meaning to those materials. It is taken with reference to a rectangular section and expresses the ratio of the modulus of rupture of a beam of given section to that of a beam of rectangular section. In the discussion on plastic bending, the form or shape factor was an expression of the ratio of the resisting moment of a flanged beam subjected to a given extreme fibre stress, to the resisting moment if this stress were uniformly distributed over the section.

Summing up rules for working stresses for brittle materials under steady loads :—

(*a*) Brittle materials in tension, bending or torsion

$$n = \frac{f_u \times F_R}{f} \qquad (7)$$

where f_u is the ultimate strength in tension, F_R is the rupture factor.

(*b*) Brittle materials in direct compression or direct shear

$$n = f_u/f \qquad (8)$$

where f_u is either the ultimate strength in direct compression when elastic instability is ruled out, or the ultimate stress in direct shear, which for cast iron is higher than the ultimate strength in tension.

In brittle materials under static loading the effect of plastic flow is small, and high local stresses are much more serious. The factor of stress concentration and brittleness of the material cannot, however, be used to predict rupture.

The ratio of the calculated stress at rupture of a plain specimen to the calculated stress at rupture for the specimen containing the stress raiser is used as a guide to the strength reducing effect of stress concentration. The ratio of these stresses is called the factor of stress concentration at rupture. One or both of these stresses may be fictitious because of the non-linear stress-strain relationship at failure. The rupture factor is only of importance in considering working stresses in brittle materials.

BIBLIOGRAPHY.

MacKenzie, J. T., and Donoho, C. K., 1937, Proc. Am. Soc. Testing Materials, Vol. 37, Part II, *A Study of the Effect of Span on the Transverse Test Results of Cast Iron.*

Salmon, E. H., *Materials and Structures* (Longmans, Green), 1931, Vol. 1, *The Elasticity and Strength of Materials*, p. 454 and p. 462.

Roark, R. J., Hartenberg, R. S., and Williams, R. Z., 1938, Eng. Exp. Sta. Univ. Wis., Bull, 82, *The Effect of Form and Scale of Strength.*

CRITERIA OF FAILURE UNDER COMBINED STRESSES.

Under simple uniaxial tension a ductile material is usually considered to have failed when marked plastic deformation has begun. For a brittle material which exhibits little ultimate elongation, rupture is the criterion.

For any state of stress in a body there are three planes at right angles on which the stress is wholly normal. These three normal stresses are called the principal stresses. One of them is the maximum stress which exists in the material, another is the corresponding minimum stress. The effect of the other principal stresses on the magnitude of the greatest principal stress at elastic breakdown has been the subject of a number of theories.

Five of these theories have received the widest acceptance :—

(1) Maximum principal stress theory (attributed to Lamé & Rankine).

Elastic failure occurs under a state of complex stress when the maximum principal stress becomes equal to the elastic limit stress.

(2) Maximum principal strain theory (St. Venant).

Elastic failure occurs when the maximum principal strain becomes equal to the strain corresponding to the elastic limit stress.

(3) Maximum shear stress theory (Coulomb & Guest).

Elastic failure occurs when the maximum shear stress is reached, *i.e.*, when the difference between the maximum and minimum principal tensile stress becomes equal to the elastic limit stress. Under tri-dimensional stress conditions, the stress of intermediate value is assumed not to affect the result.

(4) Maximum strain energy theory (Haigh).

Elastic failure occurs when the principal stresses f_x, f_y, f_z satisfy the equation

$$(f_x^2+f_y^2+f_z^2)-2/m\ (f_xf_y+f_yf_z+f_zf_z) = f_{eL}^2 \qquad (9)$$

where f_{eL} is the elastic limit stress and $1/m$ Poisson's ratio.

(5) Maximum shear-strain energy theory (Huber, Mises & Hencky), also called theory of constant energy of distortion.

Elastic failure occurs when the principal stresses f_x, f_y, f_z, satisfy the equation

$$(f_x^2+f_y^2+f_z^2)-(f_xf_y+f_yf_z+f_zf_x) = f_{eL}^2 \qquad (10)$$

A considerable amount of experimental and theoretical work is now available and the practical choice for ductile materials has narrowed down to the maximum shear stress theory and the maximum shear-strain energy theory. Moreover, both equations lead to nearly the same result and since the maximum shear stress theory is simpler in application it is more widely used as a basis for design.

In most cases the elastic limit stress is so nearly equal to the proportional limit stress that no distinction need be made between the two.

For design purposes the proportional limit stress or elastic limit stress is advisably taken as the yield stress of the material.

General case of combined stress

$$n = f_y/2q \text{ max. (Guest).} \qquad (11)$$

$$\text{or } n = f_y/[(f_x^2+f_y^2+f_z^2)-(f_xf_y+f_yf_z+f_zf_x)]. \quad \text{(Huber, Mises \& Hencky)} \qquad (12)$$

Tension and shear combined

$$n = f_y/\sqrt{(f^2+4q^2)} \qquad (13)$$

For brittle materials, as already stated, rupture constitutes the criterion, and this takes place either through tensile fracture when the maximum tensile stress reaches the ultimate stress, or through an apparent shear fracture when the maximum compressive stress reaches the ultimate stress.

The maximum principal stress and the maximum principal strain theory afford a satisfactory criterion for design.

The preceding theories of failure apply to elastic stresses. They cannot be used to predict the load carrying capacity of a machine member of ductile material under complex stresses at failure. Plastic deformation of such a member under biaxial or triaxial tensile stress is, however, much less than under uniaxial stress.

BIBLIOGRAPHY.

Guest, J. J., 1940, Proc. Inst. Automobile Eng., Vol. 35, p. 33, *Recent Research on Combined Stress.*

Salmon, E. H., *Materials and Structures* (Longmans, Green), 1931, Vol. 1, *The Elasticity and Strength of Materials*, p. 485.

Warnock, F. V., *Strength of Materials* (Pitman), 1945, p. 40 and p. 235.

Nadai, A., 1933, Trans. A.S.M.E., p. A.P.M. 55-15, Jl. Appl. Mech., Vol. 1, No. 3, *Theories of Strength.*

ELASTIC INSTABILITY.

Buckling of Compression Members.

In a member subjected to compressive stress, the load the member will sustain is determined in certain cases, not by the strength of the member but by its stiffness. This condition arises when the load produces a bending moment proportional to the corresponding deformation.

A straight slender column, when loaded axially, will remain straight only under small loads. If while thus loaded it is displaced transversely by a small amount, it will straighten itself when the disturbing force is removed. For a certain value of the axial force, however, the strut is in a state of neutral equilibrium and will remain deflected after the removal of the disturbing force. A further increase in the axial load produces a state of unstable equilibrium, and any disturbing force will start a deflection in the strut, which will increase in amount until the material is overstressed due to the increasing curvature of the member. The axial load which just induces this condition of elastic instability is known as the critical load, the buckling load or the Euler load, after the Swiss mathematician who first investigated the problem of elastic stability. Any increase in the load beyond this value is usually attended by immediate collapse of the member.

Local Buckling.

If a compression member is composed wholly or in part of thin material, local buckling may occur at a unit load less than that required to cause failure of the column as a whole. When such local buckling occurs at a stress less than the yield stress, it represents elastic instability. The critical stress at which this occurs can usually be determined mathematically.

Thin walled members, liable to local buckling, are usually stiffened by attached parts. Due to the support afforded by such parts, elastic buckling is not necessarily attended by failure, and the ultimate load that can be carried is often greater than the load producing elastic instability.

In the design of compression members, the critical load either from the consideration of the Euler column or from local instability becomes the criterion of design.

Professor Perry's formula, now generally accepted in the design of struts in steel structures, has been used to illustrate the difference in the application of a factor of safety and a load factor.

The formula is :—

$$N\,F_a = \frac{f_y + (\eta + 1) f_e}{2} - \sqrt{\left[\frac{f_y + (\eta + 1) f_e}{2}\right]^2 - f_y f_e} \qquad (14)$$

where N = load factor.
F_a = the permissible average stress.
f_y = the minimum yield stress.
f_e = Eulerian value = $\dfrac{\pi^2 E}{[l/r]^2}$
E = Young's modulus of elasticity.
η = $0{\cdot}003\, l/r$ (the coefficient depending on the material).
l/r = slenderness ratio = effective length/radius of gyration.

Using a load factor $N = 2$ and all other values as recommended by C.P. 113. Fig. 8 has been plotted for structural steel to B.S. 15. The allowable axial compressive stress is shown as ordinate and the l/r, the slenderness ratio, as abscissa.

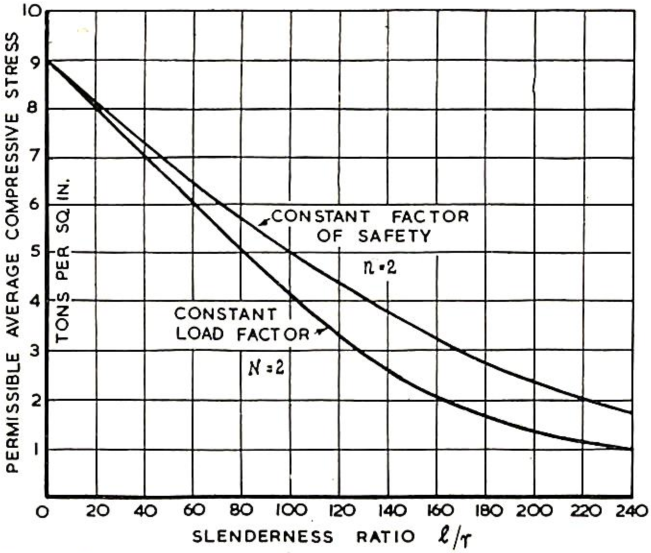


Fig. 8—Compressive Stress-Slenderness Ratio Curve for Constant Factor of Safety and Load Factor.

Equation (14) can be written in the form

$$f_1 = F_a \left(1 + \frac{\eta f_e}{f_e - F_a}\right) \tag{15}$$

where f_1 is the maximum fibre stress produced by the average compressive stress F_a. It can be shewn that a strut displaced laterally is subjected to a bending moment given by the axial compressive force and the deformation. f_1 is the sum of the resulting average compressive stress and the extreme fibre stress due to flexure. The factor of safety is then f_y/f_1. By substituting $f_y/_n$ for f_1 and solving for F_a we have :—

$$F_a = \frac{\frac{f_y}{n} + (\eta + 1) f_e}{2} - \sqrt{\left[\frac{\frac{f_y}{n} + (\eta + 1) f_e}{2}\right]^2 - \left[\frac{f_y}{n} f_e\right]} \qquad (16)$$

where n is a factor of safety.

A second curve is superimposed on Fig. 8 which has been plotted from the same formula, when the factor of safety is kept constant. It demonstrates that the use of the factor of safety leads to a considerable error for high values of slenderness ratios.

In selecting a suitable margin of safety from the consideration of elastic instability, it should be remembered that all purely theoretical formulae for critical loads are based on assumptions as to homogeneity of material, regularity of form, and boundary conditions that are never realised in practice. The critical stress to be expected under any actual set of circumstances is nearly always less than that indicated by the corresponding theoretical formula. Experimentally determined coefficients are introduced into the Rankine-Gordon, Johnstone's parabolic and Fidler's formula for struts, as well as Perry's criterion, to allow for the departure from theoretical conditions.

BIBLIOGRAPHY.

Morley, A., *Theory of Structures* (Longmans, Green), 1946, p. 281.

Pippard, A. J. S., and Baker, J. F., *The Analysis of Engineering Structures* (Edward Arnold), 1948, p. 124.

Sechler, E. E., and Dunn, L. G., *Airplane Structural Analysis and Design* (John Wiley), 1942, p. 154.

Timoshenko, S., *Strength of Materials*, Vol. 1 (Van Nostrand), 1930, p. 191.

Timoshenko, S., *Theory of Elastic Stability* (McGraw-Hill), 1936.

The Structural Use of Steel in Buildings, B.S.I., C.P. 113 (1948), General Series, p. 17.

EFFECTS OF OVERSTRAINING.

Parts of an elastic system may by accident have introduced into it initial stresses. Examples of such initial or locked up stresses are temperature stresses in welded members, the stresses in flange couplings of pressure vessels caused by screwing down the nuts, and the stresses induced in some members of a statically indeterminate structure by the tensioning of turnbuckles. Assuming that the applied loads cause stresses of like kind, for instance an additional tensile stress is applied to an initial tensile stress, the yield stress will be exceeded at a much lower imposed stress than that calculated due to the stress addition. If the load is now removed and subsequently applied again, what is the effect on the material?

Fig. 9 illustrates the effect of overstraining in tension on a ductile material, although it can be observed equally well in compression or torsion. The specimen has passed the yield stress and is in the ductile stage at A. AB is the unloading line. When applying the load again, the stress-strain relation follows BC. At C there is a sharp inflexion in the graph, CD following the path of a specimen loaded from zero to the ultimate stress. DEF shows a further process of loading and unloading.

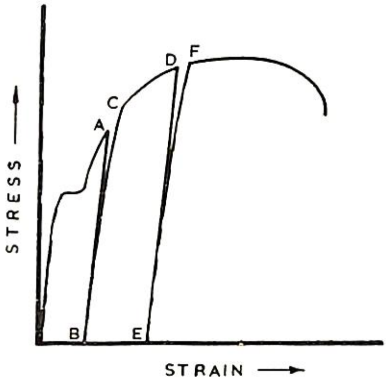


Fig. 9—Effects of Overstraining.

A change in the properties of the material takes place, which is called "hardening due to overstraining" or "strain-hardening." C and F are equivalent to the yield stress which has been raised at the expense of ductility.

In statically loaded members moderate overstressing is not a serious consideration. Under shock loading the loss in ductility, and hence reduced energy absorption capacity, must be allowed for by an increase in the factor of safety.

BIBLIOGRAPHY.

Batson, R. G., and Hyde, J. H., *Mechanical Testing* (Chapman & Hall), 1931, Vol. 1, *Testing of Materials of Construction*, p. 213.

Foster, P. F., *The Mechanical Testing of Metals and Alloys* (Pitman), 1938, p. 269.

Salmon, E. H., *Materials and Structures* (Longmans, Green), 1931, Vol. 1, *The Elasticity and Strength of Materials*, p. 469.

Warnock, F. V., *Strength of Materials* (Pitman), 1945, p. 318.

CREEP.

If materials be loaded in excess of a certain value, a gradual increase in elongation takes place with time. This phenomenon is known as creep. Most materials will creep or flow to some extent, and eventually fail under a sustained stress, less than the short-time ultimate strength. In the case of most metals, this tendency is greatly increased by high temperatures. At normal temperatures and loads, the amount of creep taking place during the service life of the part is so small that it is frequently ignored without serious consequences. Creep at high temperatures and long-time strength at atmospheric temperatures must be taken into account in the design of high-pressure steam and distillation equipment in ductile or brittle ferrous materials. Long-time strength is of importance in copper and aluminium transmission lines, lead sheathing for cables, etc.

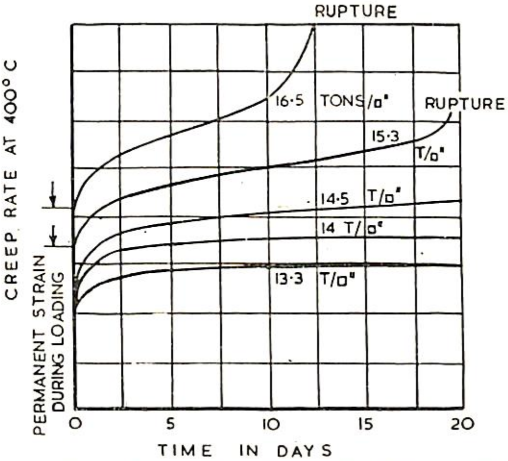


Fig. 10—Creep Rate—Time Curves for Different Values of Stress at Constant Temperature.

It is impossible within the scope of this pamphlet to lay down working rules for creep and strength diminution under long time loading, especially under high temperatures, as an appreciation of the problem requires some detailed study—generalisations are likely to prove misleading.

Creep tests provide the data necessary for determining a working stress which, at a given temperature, will produce a creep rate not higher than permissible. Fig. 10 shows a typical creep diagram for a given temperature. Creep strain is plotted as ordinate against time as abscissa for various values of stress. Interpretations of such data for design will be found in the bibliography.

BIBLIOGRAPHY.

Bailey, R. W., 1935, Proc.I.Mech.E., Vol. 131, p. 131, *Utilisation of Creep Test Data in Engineering Design.*

Batson, R. G., and Hyde, J. H., *Mechanical Testing* (Chapman & Hall), 1931, Vol. 1, *Testing of Materials of Construction*, p. 373.

McVetty, P. G., 1934, Proc. Am. Soc. Testing Materials, Vol. 34, Part II, p. 105, *The Interpretation of Creep Tests.*

Nadai, A., *Plasticity*, Eng. Societies Monograph (McGraw-Hill), 1931.

Roberts, A. M., *Mechanical Tests for Engineering Materials* (A.E.S.D.), 1948-49, p. 62.

Salmon, E. H., *Materials and Structures* (Longmans, Green), 1931, Vol. 1, *The Elasticity and Strength of Materials*, p. 579.

Sturm, R. G., Dumont, C., and Howell, F. M., 1936 Trans. A.S.M.E., Vol. 58, p. A63, *A Method of Analysing Creep Data.*

Soderberg, C. R., 1936 Trans. A.S.M.E., Vol. 58, p. R.P. 58-15, *The Interpretation of Creep Tests for Machine Design.*

Tapsell, H. J., and Johnson, A. E., 1940, Proc.I.Mech.E., Vol. 144, p. 97, *Properties cf Steels as a Basis for Design for High-Temperature Service*, Part I, *Carbon Steels.*

Warnock, F. V., *Strength of Materials* (Pitman), 1945, p. 366.

PART II.

THE CHOICE OF WORKING STRESSES UNDER DYNAMIC LOADING.

Dynamic loading can be defined as any loading during which the parts of the body cannot be considered in equilibrium, the rate of change of momentum, acceleration, must be taken into account.

Where the parts of a system have definite accelerations imposed upon them, corresponding to controlled motion, the system from the design point of view can be considered in equilibrium. This is achieved by equating the effective forces acting on the body to the inertia forces due to the configuration of the body. The system may then be thought of as loaded by these inertia forces, which are treated for the purposes of stress analysis as though they were ordinary static loads.

In impact, the loaded member is usually required to absorb a definite amount of energy. This may appear as deformation of the body in the form of strain energy, or as kinetic energy, if the body is set into motion as a result of impact. In practice very few examples of absolutely rigid bodies will be found, a strain energy term, therefore, enters into most equations in this field. The assumption of rigidity is, nevertheless, made often for convenience in mathematical computations.

When impact takes place, and for some time afterwards, the body is usually in a state of vibration, static equilibrium is only restored when the kinetic energy possessed by the impulsive force at the moment of contact has been absorbed and stored up as strain energy or dissipated as heat.

If it is assumed that the stresses due to impact are distributed throughout any elastic body exactly as in the case of static loading, then it can be shown that the ratio of impact stress to static stress, produced by the same force, is given by :—

$$f_i/f = 1 + \sqrt{(1 + 2\,h/y)} \tag{17}$$

Where f_i is the stress produced by the vertical impact of a weight falling from a height h.

f is the stress produced by the weight of the same body applied as a static load.

y is the deformation produced by the weight under static conditions.

If $h = 0$, we have the case of sudden loading and the stress produced is twice as large. It occurs when a dead load is not in motion but just in contact with a structure or machine member, its weight being supported by some means. If the load is now released suddenly, the stress and deformation produced in the reactive member are twice as large as if the load were eased on to the member as in static loading. Deformation of the supports and the weight itself would reduce the doubling effect. The actual magnitude of the impact stresses can be accurately determined only in very simple cases. It is, therefore, necessary to rely on experimental and service data in modifying the value of the ratio f_i/f.

Equation (17) also represents the ratio of impact deformation to static deformation (y).

If the impact is horizontal instead of vertical, the equation becomes :—

$$f_i/f = \sqrt{(v^2/384{\cdot}6y)} \tag{18}$$

where v is the velocity of impact in inches per second and y (in inches) as before.

Experiments on impact loading of medium carbon steels have shown that the value of the yield stress is dependent on the rate of straining. Fig. 11 shows the results of experiments in which an attempt has been made to express the ratio of dynamic to static stress for a particular kind of steel of about 20 tons yield in terms of rate of strain. The figures were roughly proportional to the rate of strain during impact.

It has been observed in experiments on medium carbon steels that under dynamic loading the upper yield stress (indicated by C in Fig. 1) is more pronounced. It may be of very high value,

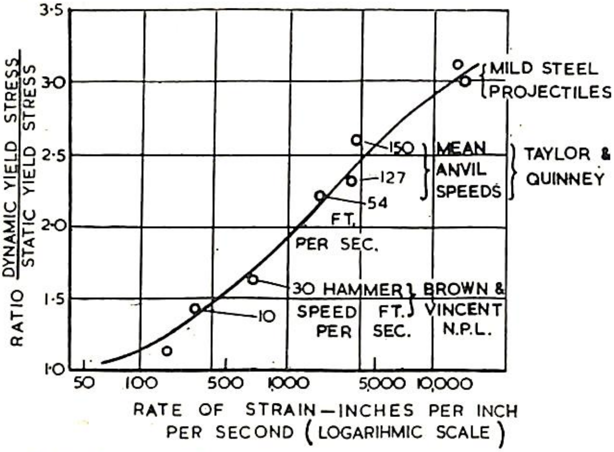


Fig. 11—Ratio Dynamic/Static Yield Stress for Varying Rates of Strain.

but is of extremely short duration, and the resistance indicated by it will not be repeated on subsequent loading. The dynamic yield stress represents a substantial increase on the static yield value (shown by B in Fig. 1), and it will be sustained as long as the material remains partially yielded. This dynamic yield stress can be utilised for design purposes if small amounts of permanent set are acceptable. (Warnock & Taylor).

For high velocities of impact (greater than 12 ft. per second), the impact yield strength decreases as the deformation continues under repeated impact, which means that should a member of a structure be slightly deformed under impact, its ability to resist further deformation due to impact is reduced. This is the reverse of the case obtaining under static loading. (Warnock & Pope).

Deformation due to impact does not raise the static yield strength appreciably. Therefore, should a member of a structure be deformed due to impact, its load-carrying capacity will not be increased as would be the case with static deformation.

Not all investigators agree that the observed increased resistance to deformation under impact loading (*i.e.*, dynamic yield stress) may be utilised for design.

It does not follow that a corresponding increase would be observable in high alloy steels, or even in similar steels under different heat treatment.

BIBLIOGRAPHY.

Brown, A. F. C., and Edmonds, R., 1948, Proc.I.Mech.E., Vol. 159, p. 11, *Thr Dynamic Yield Strength of Steel at an Intermediate Rate of Loading.*

Brown, A. F. C., and Vincent, N. D. G., 1941, Proc.I.Mech.E., Vol. 145, p. 126, *Relationship Between Stress and Strain in the Tensile Impact Test.*

Clark, D. S., and Dätwyler, G., 1938, Proc. Am. Soc. for Testing Materials, Vol. 38, Part II, p. 98, *Stress-Strain Relations Under Tension Impact Loading.*

Ginns, D. W., 1937, Journal Inst. Metals, Vol. 61, No. 2, p. 61, *The Mechanical Properties of Some Metals and Alloys Broken at Ultra-High Speeds.*

Morrison, J. L. M., 1934, *The Engineer*, Vol. 158, p. 183, *The Influence of Rate of Strain in Tension Tests.*

Warnock, F. V., and Brennan, J. B., 1943, Jl. Iron and Steel Inst., Vol. 148, p. 117, *Some Tensile Shock Properties of Carbon Steels.*

Warnock, F. V., and Brennan, J. B., 1948, Proc.I.Mech.E., Vol. 159, p. 1, *The Tensile Yield Strength of Certain Steels under Suddenly Applied Loads.*

Warnock, F. V., and Pope, J. A., 1947, Proc.I.Mech.E., Vol. 157, p. 33, *The Change in Mechanical Properties of Mild Steel Under Repeated Impact.*

Warnock, F. V., and Taylor, D. B. C., 1949, Proc.I.Mech.E., Vol. 161, p. 165, *The Yield Phenomena of a Medium Carbon Steel Under Dynamic Loading.*

IMPACT VALUES.

Dynamic determination of yield stress requires elaborate instruments and very careful experimental procedure. Moreover, it has become possible only within the last few years. Notched bar tests have been employed commercially for a very long time to obtain a measure of the behaviour of metals under dynamic load, although its value in this respect is dubious. In these tests, a specimen in the form of a notched bar is tested as a beam under impact from a hammer or pendulum. The test indicates not so much the resistance of the material to shock or impact, but rather differences of condition in the material which are brought to light through the concentration of stress occurring at the notch and which other tests fail to demonstrate. The notch localizes the stress and determines the behaviour of the material. What is recorded as having been absorbed in breaking the test piece can be divided into that required :—

(1) to deform the material up to formation of a crack.
(2) to propagate this crack.
(3) to produce stress waves in specimen and machine, including its own holding down bolts, and
(4) to impart kinetic energy to the "broken half" of the specimen.

The Izod value, as the energy measured by the Izod testing machine is referred to, and similar notched bar test values, indicate therefore energy consumed in several different ways.

Notched-bar value, as shown by the usual tests, may have a very wide range in practice without affecting reliability and utility. It is extremely rare to find examples of cases of failure in service where the Izod test figure was over 5-11 ft. lbs., where breakdown could be suggested as due to low impact value. On the other hand, if the conditions exist for the propagation of a sudden and extensive crack, a high notch bar value seems wholly unable to prevent its travel. In practice it will be found that few, if any mechanisms or structures are built of components whose materials have even approximately equal notched-bar values.

Although the practical bearing of the notched-bar value upon reliability is limited, it is generally conceded that the condition of a steel giving a high value is better than that of similar steel which gives a low one.

The principal value of the notched-bar test is now thought to lie in an indication whether the heat treatment of a steel is satisfactory.

BIBLIOGRAPHY.

Batson, R. G., and Hyde, J. H., *Mechanical Testing* (Chapman & Hall), 1931, Vol. 1, *Testing of Materials of Construction*, p. 325.

Roberts, A. M., *Mechanical Tests for Engineering Materials* (A.E.S.D.), 1948-49, p. 42.

Salmon, E. H., *Materials and Structures* (Longmans, Green), 1931, Vol. 1, *The Elasticity and Strength of Materials*, p. 515.

Warnock, F. V., *Strength of Materials* (Pitman), 1945, p. 338.

ALTERNATING STRESSES.

Practically all materials will rupture under a stress far below the ultimate strength as determined by a single load application, if this stress is repeated a number of times. This phenomenon is known as fatigue. At one time it was believed that fatigue in metals was caused by crystallisation of the material because of the appearance of the fracture and the absence of plastic flow, even in highly ductile materials. This view has been discounted by modern investigators.

Failure is due to the progressive growth across the section of a minute crack formed in the material, as yet not fully explained.

The relation between applied stress and the number of stress repetitions required to produce rupture is represented by a curve shown in Fig. 12. This curve becomes asymtotic to the line denoting a certain constant stress, the limiting range of stress below which an indefinite number of cycles of application will not cause failure ; it is also known as the endurance range, fatigue range or fatigue limit.

This curve replotted to logarithmic scales results in a straight line relation between stress and number of stress repetitions up to the value of the fatigue limit, when an abrupt transition takes place. Fig. 13 illustrates this graph.

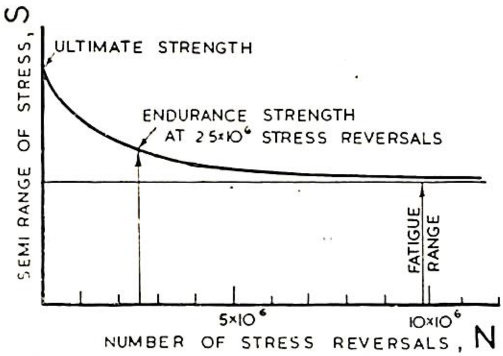


Fig. 12—Curve Showing Relation between Stress and Number of Stress Reversals to Cause Rupture.

The exponential law connecting ultimate tensile strength and fatigue range has been found to hold for a large range of ferrous metals.

In the case of non-ferrous metals there is often no well defined endurance limit, no abrupt transition taking place in the S-N diagram. Usually the number of cycles before the logarithm of stress reversals becomes constant is much greater than in wrought ferrous metals.

When using these materials and in the design of structural or machine members in metals with a definite fatigue limit where the parts need only sustain a finite number of stress reversals during their "working life," one must ascertain the highest stress at the number of cyclic reversals expected which will not cause rupture. This stress is called the endurance strength.

Fig. 14 gives some typical S-N curves for a range of engineering materials.

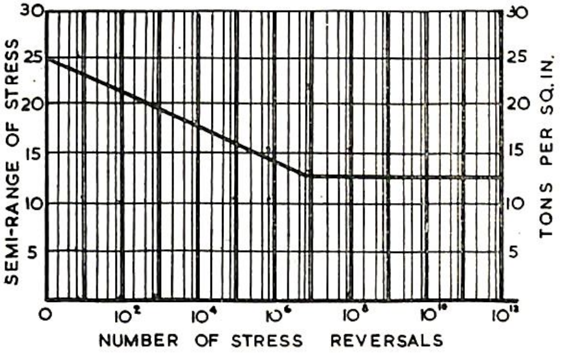


Fig. 13.—Semi-Logarithmic Plot of S/N Curve.

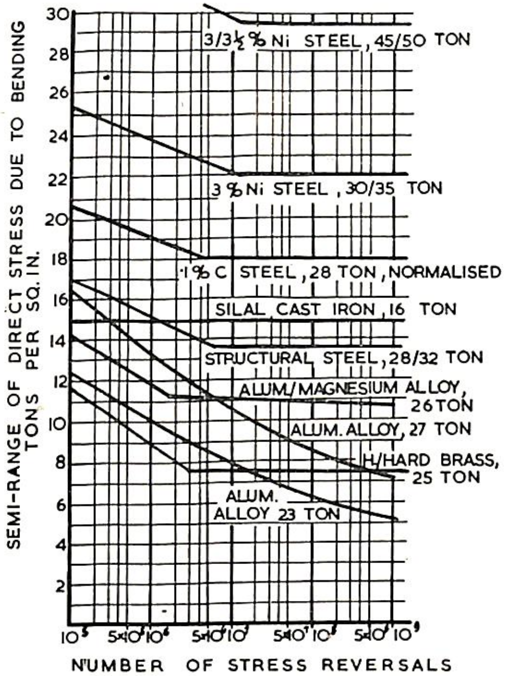


Fig. 14—S/N Curves for Various Materials.

The endurance limit may be raised slightly by producing a thin layer of high strength on the outer surface, cold rolling, burnishing or shot blasting as well as nitriding and case hardening will give this effect.

Surface imperfections by producing highly localised zones of high stress and thereby facilitating the start of minute cracks can lower the fatigue limit appreciably. Fig. 15 shows graphically the influence that mill scale, rough turning and corrosion can have on the fatigue limit. Generally the greater the ultimate strength of the material, the more severe the effect, as will be seen from the increase in the fatigue strength reduction factor. For any realised or anticipated operating conditions, this factor must be used to arrive at the true value of the fatigue limit.

There is no definite direct relationship between tenacity, *i.e.*, ultimate tensile strength and fatigue limit, as the latter has been found to vary in materials of identical ultimate tensile strength, but differing in composition and heat treatment. Fig. 16 shows an approximate curve where fatigue limit for reversed plane-bending stresses and reversed torsional stresses has been plotted against ultimate tensile strength for ductile and brittle ferrous

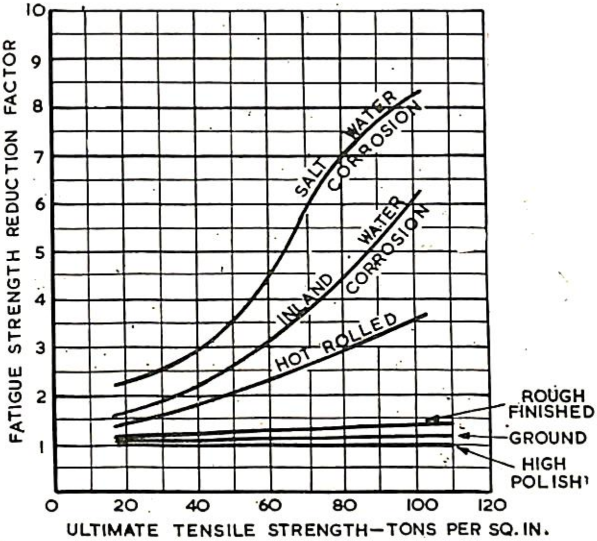


Fig. 15—Effect of Surface on Fatigue Strength Reduction.

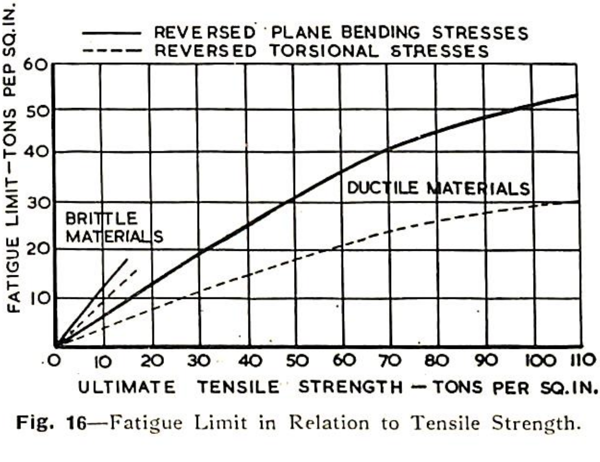


Fig. 16—Fatigue Limit in Relation to Tensile Strength.

materials. In view of what has been said previously, the graph should be used only with great caution for design purposes.

It will be seen that the slope gradually falls as the ultimate tensile strength increases. 0·5 f_u appears to be a limiting value for the fatigue limit in reversed bending (b), 0·27 f_u for reversed torsional stresses (t). Although the lower curve reproduces the characteristics of the upper one, again no direct definite relationship between b and t can be established. The ratio b/t has been found to lie from 0·4 to 0·8.

The endurance limit does not seem to be altered at stress repetitions below 5,000 cycles per minute. For higher frequencies, in general, the fatigue range increases to an extent dependent on the material. With many metals and alloys, the increase is appreciable when frequencies of the order of 10,000 cycles per minute are reached. (Gough).

In design of machine or structural members subjected to cyclic variations of stress, the fatigue strength or endurance strength becomes now the criterion of failure and the factor of safety is respectively

$$n = \frac{b}{\pm f} \text{ for reversed plane bending stresses} \qquad (19)$$

$$\text{and } n = \frac{t}{\pm q} \text{ for reversed torsional stresses} \qquad (20)$$

where b is the fatigue limit under reversed bending stresses
t is the fatigue limit under reversed shearing stresses.
f is the applied reversed bending stress.
and q is the applied reversed torsional stress.

COMBINED ALTERNATING BENDING AND SHEARING STRESS.

Assuming the conditions for the separately acting cyclic stresses to be fully determinate, the simultaneous applications of these stresses raises an important problem.

Plotted in the manner shown in Fig. 17, the relation between f and q corresponding to failure is a curve passing through t and b, but always falling outside the straight line connecting these points. In using this line as the criterion of failure, the design would err on the safe side but might lead to uneconomical proportions. Experimental material available does, however, permit certain conclusions.

For ductile materials the combination of f and q, which will cause failure can be represented by an ellipse quadrant, given by :—

$$f^2/b^2 + q^2/t^2 = 1 \qquad (21)$$

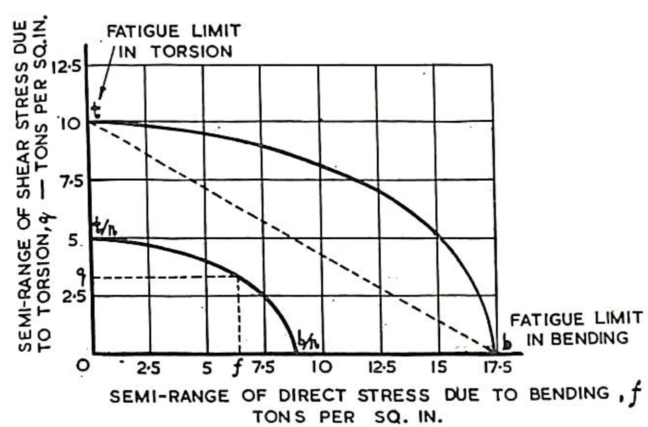


Fig. 17—Typical f/q Curve.

For brittle materials the ellipse arc is more appropriate, the expression being :—

$$q^2/t^2 + f^2\,(b/t-1)/b^2 + f\,(2-b/t)/b = 1 \qquad (22)$$

Both curves pass through points t for $f = 0$ and b for $q = 0$.

In order to obtain a factor of safety, the stresses for ductile materials must follow the equation :—

$$f^2/b^2 + q^2/t^2 = 1/n^2 \qquad (23)$$

representing an ellipse quadrant connecting the points t/n_s for $f=0$ and b/n_b for $q = 0$. It is evident that $t/q = n_s$ and $b/f = n_b$ represent the factor of safety when these stresses are acting alone, hence the result may be written :—

$$1/n_s^2 + 1/n_b^2 = 1/n^2 \qquad (24)$$

The application of the maximum shear theory to combined variable stresses also is not appropriate, in view of the wide limit of the b/t ratio. In many cases it would lead to a conservative design, but for values of b/t less than 0·5 proportions would be inadequate.

COMBINED STEADY AND VARIABLE STRESS.

The general case of a steady and variable stress can be represented as in Fig. 18. The minimum stress during one cycle is given by $f_s - f$ and the maximum stress by $f_s + f$.

If the steady stress is plotted as abscissa against the variable stress as ordinate, the experimental curve will take the shape shown by the dotted line, Fig. 19. In general this line varies both with the material and the type of applied stress. With increasing values of static stress the fatigue limit diminishes in general far

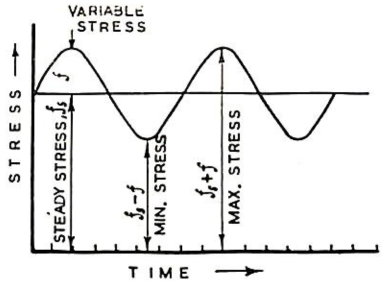


Fig. 18—Typical Combined Variable and Steady Stress/Time Curve.

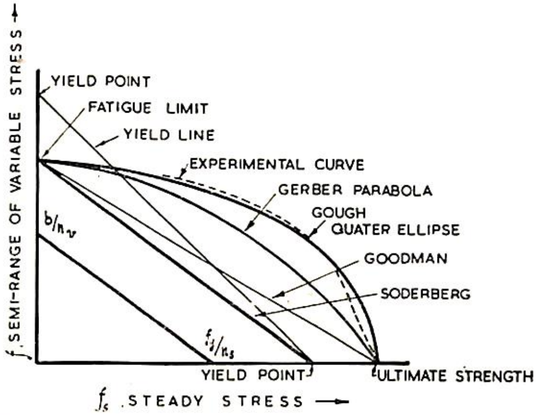


Fig. 19—Combined Steady and Variable Stress Curves.

more rapidly for soft, ductile non-ferrous metals than with steel, and much more sharply under direct stresses than with flexural or torsional stresses for most metals and alloys. (Gough).

Gerber approximated this curve as a parabola, while Goodman approximated it as a straight line between the endurance limit and the ultimate stress. Gough suggests that the ellipse quadrant is probably of general application for ductile materials. In view of the somewhat limited range of available experimental evidence and the many variables involved, it appears reasonable to treat the problem on the basis that the straight line is the actual line of failure, although there remains in certain cases a considerable field outside this relationship to be utilised.

The fatigue line cuts off a part of the right angled triangle that would enclose all safe combinations of stress if plastic yield alone were in question, and the safe zone in the diagram is seen to be bounded partly by the fatigue line and partly by the yield line.

Soderberg has suggested the use of the straight line connecting endurance limit and yield stress as the criterion for combinations of steady and variable stress, which will cause failure. The equation of this line is :—

$$(f_s/f_y) + (f/b) = 1 \tag{25}$$

The design stress line is then expressed by the straight line relation

$$(f_s/f_y) + (f/b) = 1/n \tag{26}$$

where n is a factor of safety. When these stresses are acting alone, the factors of safety are given by $f_y/f_s = n_s$ and $b/f = n_v$ and $1/n = (1/n_s) + (1/n_v)$. (27)

Where n_s is the factor of safety applied to the steady stress.
and n_v ,, ,, ,, ,, ,, ,, variable stress.
Equation 26 may be written in the form

$$1/n = [f_s + f(f_y/b)]/f_y \tag{28}$$

In which form it expresses the fact that a variable stress can be added to the steady stress by weighting it with the factor fy/b.

Equation 26 is equally valid for a steady torsional stress, superimposed on a variable stress of like nature. Considering ductile materials and applying the maximum shear stress theory, the yield stress in torsion will be given by $fy/2$, for the maximum shear energy theory it will be $fy/\sqrt{3}$ and the equations are respectively :—

$$(2q_s/f_y) + (q/t) = 1/n \tag{29}$$

$$\text{or } (\sqrt{3}q/f_y) + (q/t) = 1/n \tag{30}$$

COMBINATION OF STATIC AND VARIABLE BENDING AND TORSIONAL STRESSES IN DUCTILE MATERIALS.

Fig. 20 shows in quadrant 1 the relationship between steady and variable bending stresses as defined by the Soderberg straight line. f_y is the yield stress in bending or direct tension of the material and b the fatigue limit under reversed bending stresses. Quadrant 3 represents the relationship between steady and variable torsional stresses· Quadrant 2 defines the combination of variable bending and torsional stresses acting simultaneously. f_{ys} is the yield stress in torsion of the material and t the fatigue limit under reversed torsional stresses. The curve b-t is the ellipse quadrant. Quadrant 4 is the case of steady bending and torsional stresses acting together. The equation for this line is given by :—

$$(f_s/f_y)^2 + (2q_s/f_y)^2 = 1 \text{ (maximum shear theory)} \tag{31}$$

representing the quarter ellipse that gives failure through yielding.

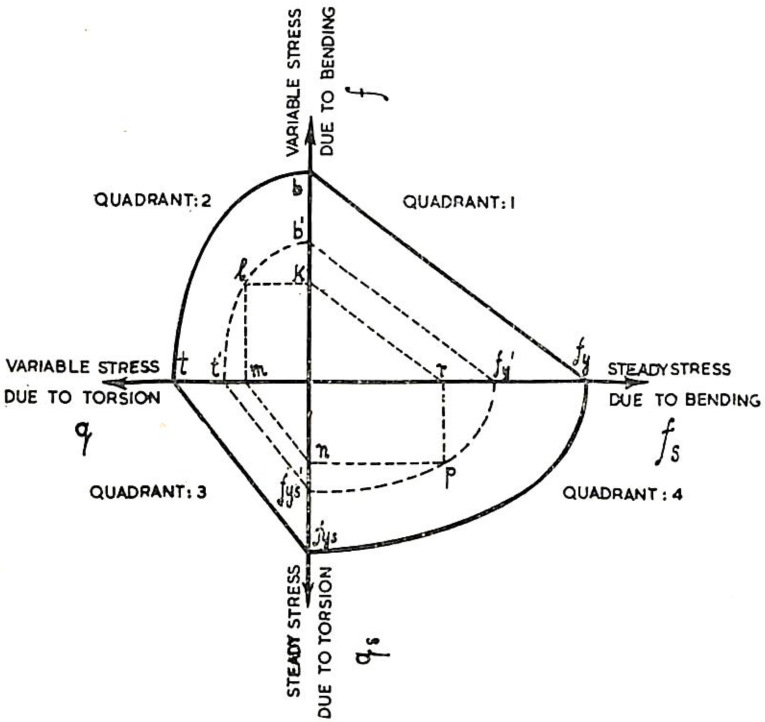


Fig. 20—Conditions of Failure for Steady and Variable Bending and Torsional Stresses Acting Simultaneously in Phase.

Applying the same factor of safety to the steady and alternating components, the lines connecting $b^1f_y{}^1$, $t^1f_{ys}{}^1$, t^1b^1 and $f_y{}^1f_{ys}{}^1$, represent the design conditions for any two stresses acting simultaneously. In the general case when all the stresses are present and the variable components act in phase, it can be shown that the straight lines *kl, lm, mn, np* and *pr* represent the limits for design purposes.

Applying the maximum shear stress theory, the general equation becomes :—

$$\{ (f_s/f_y) + (f/b) \}^2 + \{ (2q_s/f_y) + (q/t) \}^2 = 1/n^2 \qquad (32)$$

In actual calculations it will often be simpler to apply a load factor, rather than a factor of safety.

BIBLIOGRAPHY.

Bailey, R. W., 1940, Proc.I.Mech.E., Vol. 143, p. 101, 1941, Proc.I.Mech.E., Vol. 145, p. 46, *Plastic Yielding and Fatigue of Ductile Metals. A Criterion and its Application.*

Batson, R. G., and Hyde, J. H., *Mechanical Testing* (Chapman & Hall), 1931, Vol. 1, *Testing of Materials of Construction*, p. 231.

Foster, P. F., *The Mechanical Testing of Metals and Alloys* (Pitman), 1938, p. 197.

Gough, H. J., and Pollard, H. V., 1935, Proc.I.Mech.E., Vol. 131, p.3, *The Strength of Metals Under Combined Alternating Stresses.*

Gough, H. J., 1949, Proc.I.Mech.E., Vol. 160, p. 417, *Engineering Steels Under Combined Cyclic and Static Stresses.*

Guest, J. J., 1940, Proc. Inst. Automobile Eng., Vol. 35, p. 33, *Recent Research on Combined Stress.*

Hohenemser, K., and Prager, W., 1933, Metallwirtschaft, Vol. 12, p. 342, *The Problem of Fatigue Strength Under Complex Stresses.*

Roberts, A. M., *Mechanical Tests for Engineering Materials* (A.E.S.D.), 1948-49, p. 50.

Salmon, E. H., *Materials and Structures* (Longmans, Green), 1931, Vol. 1, *The Elasticity and Strength of Materials*, p. 540.

Temple, J. E., *Handbook of Structural Design in the Aluminium Alloys* (James Booth & Company, Birmingham), 1948, p. 24.

STRESS CONCENTRATIONS UNDER FATIGUE CONDITIONS.

Under static loading in ductile materials one can as a rule disregard peak stresses due to stress concentration due to the fact that the stress "evens up" on local yielding.

Very different conditions exist when a machine part is subjected to cyclic reversal of stress. The fatigue crack may begin at a stress below the yield point of the material, and a full effect of the peak stress may be expected; it follows that ductility is not a measure of immunity to stress concentration in fatigue, ductile materials being, in fact, more susceptible than brittle ones.

Even the most highly localised stresses may greatly reduce the endurance limit, but materials vary greatly in "notch sensitivity," as susceptibility to this effect is sometimes called.

The weakening effect of peak stresses is much smaller in practice than considerations of the stress concentration factor for elastic stress would lead one to expect. What may be termed the factor of stress concentration for fatigue or fatigue strength reduction factor (K_f) is defined as the ratio of the fatigue strength of a solid specimen to the fatigue strength of the specimen containing the stress raiser, the latter being a nominal stress. K_f is the true measure of the weakening effect of the stress raiser under fatigue conditions.

K (the elastic stress concentration factor) can be considered as a limiting value, sometimes approached but rarely equalled by K_f.

The susceptibility to the stress concentration effect is expressed by the fatigue notch sensitivity index Q.

$$Q = \frac{K_f - 1}{K - 1} \tag{35}$$

$Q = 0$ when $K_f = 1$, the stress raiser producing no reduction in fatigue strength.

and $Q = 1$ when $K_f = K$ when the full theoretical effect is realised.

Q is not constant for either geometrically similar or even identical specimens of different materials. One might expect that the notch-bar impact values, as determined by a conventional test, would give an indication of relative fatigue strength where a sharp notch is present. This, however, is not so.

One can say that with greater size Q increases, and for very large specimens approaches unity, *i.e.*, the full theoretical weakening effect is realised. This is called the scale effect. In other words, the fatigue strength reduction factor varies with the size of the member for similar or identical stress raisers. Further, the fatigue strength reduction factor varies according to the nature of the stresses applied. Investigations have shown that K_f is also affected by the combination of dissimilar stresses.

In this connection it has been found that an ellipse arc as defined by equation 22 affords a satisfactory criterion for the resistance of ductile steels under combined bending and torsion when a stress concentration effect is present.

Under conditions of a steady and variable stress acting together the fatigue strength reduction factor need only be applied to the variable component of stress. Thus equation (25) becomes :—

$$(f/f_y) + (K_f f/b) = 1 \tag{36}$$

and the design-stress line is :—

$$(f/f_y) + (K_f f/b) = 1/n \tag{37}$$

similarly equation (32) becomes :—

$$\{ (f_s/f_y) + (K_f f/b) \}^2 + \{ (2q_s/f_y) + (K_f q/t) \}^2 = 1/n^2 \tag{38}$$

It must be realised that K_f may have a different value for flexure and torsion.

An important fact concerning stress concentration is that a single isolated notch or V-groove has a worse effect than a number of similar stress raisers placed close together. Thus, the fatigue strength reduction effect is less due to a continuous screw thread than to a single V-groove.

The practical significance of the foregoing is that the factor of safety under alternating load conditions should consist of three parts :—

(1) a factor for fatigue reduction due to stress concentration.

(2) a further factor for surface finish or surface deterioration expected, such as corrosion ; also expressed as fatigue strength reduction factor,

and (3) the real margin of safety.

The criterion of failure has now become the fatigue limit under the appropriate stress conditions, or, as previously discussed, the appropriate *b-t* relation.

Expressed in terms of working stress one can write

$$\text{Working stress} = \frac{\text{Fatigue Limit}}{\text{Stress concentration factor} \times \text{surface finish factor} \times \text{real margin of safety.}}$$

In view of the many variables involved in a true assessment of K_f, the data provided in the Appendix has been limited, to stress concentration factors for elastic stresses, and it is suggested, in cases where no experimental information can be obtained, that values of K should be used, affording a further unknown margin of safety.

BIBLIOGRAPHY.

Gough, H. J., 1949, Proc.I.Mech.E., Vol. 160, p. 417, *Engineering Steels Under Combined Cyclic and Static Stresses.*

Petersen, R. E., and Wahl, A. M., 1936, Trans. A.S.M.E., Vol. 58, p. A15, *Two and Three Dimensional Cases of Stress Concentration and Comparison with Fatigue Tests.*

Frocht, M. M., 1935, Trans. A.S.M.E., Vol. 57, p. A67, *Factors of Stress Concentration Photoelastically Determined.*

Peterson, R. E., 1932, Proc. Am. Soc. Testing Materials, Vol. 32. Part II., p. 413, *Fatigue of Shafts Having Keyways.*

Timoshenko, S., 1947, Proc.I.Mech.E., Vol. 157, p. 163, *Stress Concentration and Fatigue Failures.*

Wahl, A. M., and Beeuwkes, R., 1934, Trans. A.S.M.E., Vol. 56, p. 617, *Stress Concentration Produced by Holes and Notches.*

ACKNOWLEDGMENTS.

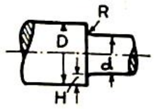
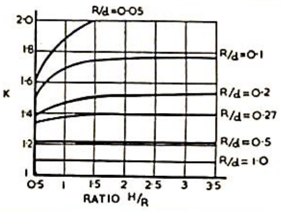
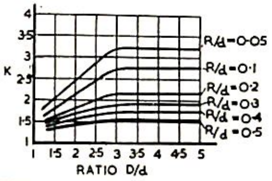
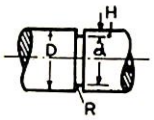
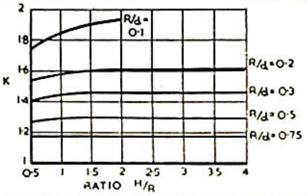
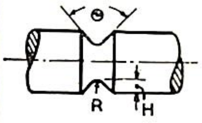
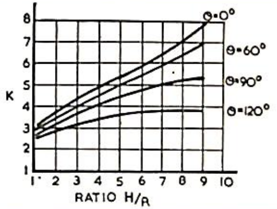
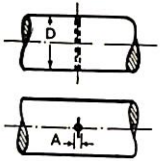
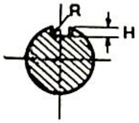
In conclusion, the author wishes to express his indebtedness to the Institution of Mechanical Engineers and the Institution of Structural Engineers for kind permission to make use of material from their *Proceedings* and *Journal* respectively.

APPENDIX I.

Values of Rupture Factor for Cast Iron.

Form of Member and Manner of Loading.	Rupture Factor, ratio of computed max. stress at rupture to ultimate tensile strength.
(1) Rectangular beam, simply supported at ends, loaded at centre span/depth ratio = 8 or more.	1·70
(2) Circular beam, simply supported at ends, loaded at centre.	2·80 — 0·025 × Ultimate Tensile strength. U.T.S. in thousands of lbs. per sq. inch.
(3) Bar of circular section in torsion.	1·02 to 1·79
(4) Bar of triangular section in torsion.	1·52
(5) Bar of rectangular section in torsion.	1·50

APPENDIX II. Factors of Stress Concentration for Elastic Stress.

Type of Stress Raiser	Manner of Loading	Factor of Stress Concentration, K
(1) Square shoulder with fillet in circular shaft 	Bending	
	Torsion	
(2) U-notch in circular shaft 	Bending	
	Torsion	$K = \dfrac{(D-d+2R)\,(d+2R)^2 + 4R^2(D-d-2R)}{2RD\,(d+4R)}$
(3) V-notch in circular shaft 	Tension, compression and bending	
(4) Radial hole in circular shaft 	Bending	$K = \dfrac{3D}{A + D}$
	Torsion	For A/D very small, K = 4
(5) Keyway in circular shaft 	Torsion	$K = 1 + \sqrt{(H/R)}$ (approximate formula for any longitudinal groove)

A.E.S.D. Printed Pamphlets and Other Publications in Stock.

An up-to-date list of A.E.S.D. pamphlets in stock is obtainable on application to the Editor, *The Draughtsman*, Onslow Hall, Little Green, Richmond, Surrey.

A similar list is also published in *The Draughtsman* twice a year.

Readers are asked to consult this list before ordering pamphlets published in previous sessions.

List of A.E.S.D. Data Sheets.

1. Safe Load on Machine-Cut Spur Gears.
2. Deflection of Shafts and Beams.
3. Deflection of Shafts and Beams (Instruction Sheet). } Connected.
4. Steam Radiation Heating Chart.
5. Horse-Power of Leather Belts, etc.
6. Automobile Brakes (Axle Brakes).
7. Automobile Brakes (Transmission Brakes). } Connected.
8. Capacities of Bucket Elevators.
9. Valley Angle Chart for Hoppers and Chutes.
10. Shafts up to 5½-in. diameter, subjected to Twisting and Combined Bending and Twisting.
11. Shafts, 5¾ to 26 inch diameter, subjected to Twisting and Combined Bending and Twisting.
12. Ship Derrick Booms.
13. Spiral Springs (Diameter of Round or Square Wire).
14. Spiral Springs (Compression).
15. Automobile Clutches (Cone Clutches).
16. ,, ,, (Plate Clutches).
17. Coil Friction for Belts, etc.
18. Internal Expanding Brakes. Self-Balancing Brake Shoes (Force Diagram).
19. Internal Expanding Brakes. Angular Proportions for Self-Balancing. } Connected.
20. Referred Mean Pressure Cut-Off, etc.
21. Particulars for Balata Belt Drives.
22. ⅞″ Square Duralumin Tubes as Struts.
23. 1″ ,, ,, ,, ,,
24. ¾″ Sq. Steel Tubes as Struts (30 ton yield).
25. ⅞″ ,, ,, ,, (30 ton yield).
26. 1″ ,, ,, ,, (30 ton yield).
27. ¾″ ,, ,, ,, (40 ton yield).
28. ⅞″ ,, ,, ,, (40 ton yield).
29. 1″ ,, ,, ,, (40 ton yield).
30. Moments of Inertia of Built-up Sections (Tables).
31. Moments of Inertia of Built-up Sections (Instructions and Examples). } Connected.
32. Reinforced Concrete Slabs (Line Chart).
33. Reinforced Concrete Slabs (Instructions and Examples) } Connected.
34. Capacity and Speed Chart for Troughed Band Conveyors.
35. Screw Propeller Design (Sheet 1, Diameter Chart).
36. ,, ,, ,, (Sheet 2, Pitch Chart).
37. ,, ,, ,, (Sheet 3, Notes and Examples) } Connected.
38. Open Coil Conical Springs.
39. Close Coil Conical Springs.
40. Trajectory Described by Belt Conveyors (Revised 1949),
41. Metric Equivalents.
42. Useful Conversion Factors.
43. Torsion of Non-Circular Shafts.
44. Railway Vehicles on Curves.
46. Coned Plate Development.
47. Solution of Triangles (Sheet 1, Right Angles).
48. Solution of Triangles (Sheet 2, Oblique Angles).
49. Relation between Length, Linear Movement and Angular Movement of Lever (Diagram and Notes).
50. ,, ,, ,, ,, ,, ,, (Chart).
51. Helix Angle and Efficiency of Screws and Worms.
52. Approximate Radius of Gyration of Various Sections.

53. Helical Spring Graphs (Round Wire).
54. ,, ,, ,, (Round Wire). Connected.
55. ,, ,, ,, (Square Wire).
56. Relative Value of Welds to Rivets.
58. Graphs for Strength of Rectangular Flat Plates of Uniform Thickness.
59. Graphs for Deflection of Rectangular Flat Plates of Uniform Thickness.
60. Moment of Resistance of Reinforced Concrete Beams.
61. Deflection of Leaf Spring.
62. Strength of Leaf Spring.
63. Chart Showing Relationship of Various Hardness Tests.
64. Shaft Horse Power and Proportions of Worm Gear.
65. Ring with Uniform Internal Load (Tangential Strain) Connected.
66. Ring with Uniform Internal Load (Tangential Stress)
67. Hub Pressed on to Steel Shaft. (Maximum Tangential Stress at Bore of Hub).
68. Hub Pressed on to Steel Shaft. (Radial Gripping Pressure between Hub and Shaft).
69. Rotating Disc (Steel) Tangential Strain. Connected.
70. ,, ,, ,, Stress.
71. Ring with Uniform External Load, Tangential Strain. Connected.
72. ,, ,, ,, ,, ,, Stress.
73. Viscosity Temperature Chart for Converting Commercial to Absolute Viscosities. Connected.
74. Journal Friction on Bearings.
75. Ring Oil Bearings.
76. Shearing and Bearing Values for High Tensile Structural Steel Shop Rivets, in accordance with B.S.S. No. 548/1934.
78. Velocity of Flow in Pipes for a Given Delivery. Connected.
79. Delivery of Water in Pipes for a Given Head.
80. (See No. 105).
81. Involute Toothed Gearing Chart.
83. Variation of Suction Lift and Temperature for Centrifugal Pumps.
89. Curve Relating Natural Frequency and Deflection.
90. Vibration Transmissibility Curved or Elastic Suspension. Connected.
91. Instructions and Examples in the Use of Data Sheets, Nos. 89 and 90.
92. Pressure on Sides of Bunker.
93-4-5-6-7. Rolled Steel Sections.
98-99-100. Boiler Safety Valves.
102. Pressure Required for Blanking and Piercing.
103. Punch and Die Clearances for Blanking and Piercing.
104. Nomograph for Valley Angles of Hoppers and Chutes.
105. Permissible Working Stresses in Mild Steel Struts to B.S. 449, 1948.

(Data Sheets are 3d to Members, 6d to others, post free).

Orders for Pamphlets and Data Sheets to be sent to the Editor, *The Draughtsman*, cheques and orders being crossed "A.E.S.D."